Solutions: 9 Ferroelectricity 14.04.2025

9.1.

The electric field and polarization are linked with the equation of state

$$E = \alpha P + \beta P^3$$

with $\alpha = A(T - T_c) = (T - T_c)/(\varepsilon_0 C_{CW})$ – see the definition of Curie-Weiss constant.

(a)

At the transition temperature, $\alpha = A(T - T_c) = 0$, and equation is reduced to

$$E_0 = \beta P_0^3$$

The polarization can be found as:

$$P_0 = \left(\frac{E_0}{\beta}\right)^{1/3} = \left(\frac{5 \times 10^5 \text{ V/m}}{4 \times 10^9 \text{ Jm}^5/\text{C}^4}\right)^{1/3} = 0.05 \text{ C/m}^2$$

The average permittivity is then:

$$K_{av} = 1 + \frac{1}{\varepsilon_0}\frac{P_0}{E_0} = 1 + \frac{1}{8.85 \times 10^{-12} \text{ F/m}}\left(\frac{0.05 \text{ C/m}^2}{5 \times 10^5 \text{ V/m}}\right) = 11300$$

(b)

When $T > T_c$ and $E_0 = 0$, then spontaneous polarization $P_0 = 0$. Differential permittivity for this case can be found as:

$$K = 1 + \frac{1}{\varepsilon_0}\frac{\partial P}{\partial E} = 1 + \frac{1}{\varepsilon_0}\frac{1}{\alpha + 3\beta P^2} = \frac{1}{\varepsilon_0 \alpha} = \frac{C_{CW}}{T - T_c}$$

Thus, when the temperature goes to the Curie temperature, $T \to T_c$, the differential permittivity tends to infinite value.

(c)

In both cases (a) and (b), measurements of permittivity of the material are made at the same temperature. However, in (a) the permittivity has a finite value, while in (b) it is clearly infinite.

These results are different since in case (a) an electric field of relatively large value was applied for measurement of permittivity, while in (b), as a definition for differential permittivity, the response of polarization was measured on application of electric field of almost-zero value.

The difference becomes clear once one investigates the ratio P_0/E_0 in case (a):

$$\frac{P_0}{E_0} = \frac{1}{\beta^{1/3} E_0^{2/3}}$$

which goes to infinity once $E_0 \to 0$.

9.2.

The electric field and polarization are linked with the equation of state

$$E_0 = \alpha P + \beta P^3,$$

with $\alpha = A(T - T_c) = (T - T_c)/(\varepsilon_0 C_{CW})$ – see the definition of Curie-Weiss constant.

In the presence of an electric field, the measured permittivity reads :

$$K(E_0) = 1 + \frac{1}{\varepsilon_0}\frac{\partial P}{\partial E} = 1 + \frac{1}{\varepsilon_0}\frac{1}{\alpha + 3\beta P^2}$$

From $E_0 = \alpha P + \beta P^3$ it is clear that with increasing field E_0 polarisation P increases as well, thus the permittivity $K(E_0)$ decreases.

When close to the phase transition, $K(E_0) \gg 1$, and

$$K(E_0) \approx \frac{1}{\varepsilon_0}\frac{1}{\alpha + 3\beta P^2}$$

When at zero electric field, $K(0)$ is measured as:

$$K(0) = \frac{1}{\varepsilon_0 \alpha}$$

Under the conditions of the problem, the value of $K(E_0)$ measured at the field E_0 must be two times smaller, $K(E_0) = \frac{1}{2}K(0)$:

$$\frac{1}{\varepsilon_0}\frac{1}{\alpha + 3\beta P^2} = \frac{1}{\varepsilon_0}\frac{1}{2\alpha}$$

$$3\beta P^2 = \alpha$$

$$P^2 = \frac{\alpha}{3\beta}$$

Substituting obtained value for polarisation into $E_0 = \alpha P + \beta P^3$:

$$E_0 = \alpha\sqrt{\frac{\alpha}{3\beta}} + \beta\frac{\alpha}{3\beta}\sqrt{\frac{\alpha}{3\beta}} = \frac{4}{3}\sqrt{\frac{\alpha^3}{3\beta}}$$

Finally, rewriting in terms of Curie-Weiss constant and temperature, $\alpha = (T - T_c)/(\varepsilon_0 C_{CW})$:

$$E_0 = \frac{4}{3}\sqrt{\frac{\alpha^3}{3\beta}} = \frac{4}{3}\sqrt{\frac{(T - T_c)^3}{3\varepsilon_0^3 C_{CW}^3 \beta}}$$

$$= \frac{4}{3}\sqrt{\frac{(10\ \mathrm{K})^3}{3 \cdot (8.85 \times 10^{-12}\ \mathrm{F/m})^3 \cdot (1.1 \times 10^5\ \mathrm{K})^3 \cdot 4 \times 10^9\ \mathrm{Jm^5/C^4}}} = 4 \times 10^5\ \mathrm{V/m}$$

$$= 4\ \mathrm{kV/cm}$$